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International Journal of Heat and Mass Transfer 46 (2003) 1607–1611
International Journal of HEAT and MASS TRANSFER
www.elsevier.com/locate/ijhmt

Thermal waves propagation in porous material undergoing thermal loading

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Received 18 September 2002

Abstract

A model for predicting thermal waves within a surface-heated porous structure has been developed. The relevant phenomena for the moisture, pressure and temperature fields are coupled. Considering mass and energy transfer processes, a set of governing differential equations is presented. The solution of the problem has been obtained with a finite difference scheme.

1. Introduction

Thermal waves in porous media have recently received growing attention in light of the common usage of such media in various applications in the fields of energy technology.

The theoretical analysis of heat and moisture transport in porous material relies upon the underlying heat flux and moisture transfer constitutive models and the fusion of the mechanisms associated with heat conduction to those of poroelasticity lead to the fundamental description of the thermo-poro-elastic behaviour of solids.

The physical phenomena of moisture transfer in porous media are usually explained by diffusion theory, capillary flow theory and evaporation condensation theory [1–24].

In a present work the evaporation mechanism was assumed with concentration and pressure gradient terms.

Among the various currently available heat flux constitutive models, the Fourier model [25] which is based upon steady-state assumptions, has long been widely accepted for a variety of practical engineering situations. With the Fourier model the resulting transient temperature equations are of the parabolic type.

To account for the temperature propagation speed, and to account for any anomalies associated with the Fourier model, the Cattaneo model [26] was introduced. With the Cattaneo model, the resulting temperature equations are of the hyperbolic type. The Cattaneo and a Fourier-like diffusive models are subcases that can be degenerated from the so-called Jeffrey'-type model.

Additionally, a variety of misconceptions and perceptions exist in the literature, including particular attempts to relate macroscale in space phenomenon in explaining the underlying microscale in space heat-conduction phenomena in pulse heating, nonphysical bounds, violation of causality principles and the like, thereby altering the fundamentals associated with the heat transport characteristics.

In the present article Cattaneo-type heat conduction model is assumed to account for the temperature propagation speed. The above model may be use to formulated generalized theory of dynamic thermo-poro-elasticity.

2. Governing equations

2.1. Conservation equations for mass

The conservation equations for mass can be written as

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0017-9310/03/$ - see front matter
PII: S0017-9310(02)00437-4

$$\dot{\rho}_i = -\nabla(\rho_i w_i) + W_i, \tag{1}$$

where ρ_i is the density of species i, w_i is the velocity of species i and W_i is the production rate of species i. Since no movement of the liquid (subscript l) is assumed $w_\mathrm{l} = 0$. Also, $W = -W_\mathrm{v} = -W_\mathrm{m}$ (v = vapor, m = air–vapor mixture), since the rate of liquid evaporation is the same as the rate of vapor production, and since the air does not change phase, the rate of mixture production equals the rate of vapor production. The above equation thus simplifies to

$$\dot{\rho}_\mathrm{l} = -W_\mathrm{m}. \tag{2}$$

The continuity equation for the air (subscript a) takes the form

$$\dot{\rho}_\mathrm{a} = -\nabla(\rho_\mathrm{a} w_\mathrm{a}) \tag{3}$$

and for the vapor,

$$\dot{\rho}_\mathrm{v} = -\nabla(\rho_\mathrm{v} w_\mathrm{v}) + W_\mathrm{v}. \tag{4}$$

The conservation of gas phase mass gives

$$\dot{\rho}_\mathrm{m} = -\nabla(\rho_\mathrm{m} w_\mathrm{m}) + W_\mathrm{m}. \tag{5}$$

Fick's law allows the fluxes to be presented in the forms of Eqs. (6) and (7):

$$j_\mathrm{a} = \rho_\mathrm{a}(w_\mathrm{a} - w_\mathrm{m}) = -\rho_\mathrm{m} D \nabla \rho_{\beta\mathrm{a}}, \tag{6}$$

where $\rho_{\beta i} = \rho_i/\rho_\mathrm{m}$ is the mass fraction of species i with respect to the density of the air–vapor mixture, and D is the diffusion coefficient for Fick's law for the air–vapor mixture; and

$$j_\mathrm{v} = \rho_\mathrm{v}(w_\mathrm{v} - w_\mathrm{m}) = -\rho_\mathrm{m} D \nabla \rho_{\beta\mathrm{v}}. \tag{7}$$

Finally, we get the following species equations:

$$\rho_\mathrm{m}\dot{\rho}_{\beta\mathrm{a}} + \rho_\mathrm{m} w_\mathrm{m} \nabla \rho_{\beta\mathrm{a}} = \nabla\left(\rho_\mathrm{m} D \nabla \rho_{\beta\mathrm{a}}\right) - \rho_{\beta\mathrm{a}} W_\mathrm{m} \tag{8}$$

and

$$\rho_\mathrm{m}\dot{\rho}_{\beta\mathrm{v}} + \rho_\mathrm{m} w_\mathrm{m} \nabla \rho_{\beta\mathrm{v}} = \nabla\left(\rho_\mathrm{m} D \nabla \rho_{\beta\mathrm{v}}\right) + \left(1 - \rho_{\beta\mathrm{v}}\right) W_\mathrm{m}. \tag{9}$$

2.2. Thermal equations

The fluxes of heat q and flowing gases r can be expressed by the Cattaneo-type model as

$$\tau \frac{\partial q}{\partial t} = -q - \lambda \nabla \theta \tag{10}$$

and

$$r = \rho_\mathrm{a} w_\mathrm{a} h_\mathrm{a} + \rho_\mathrm{v} w_\mathrm{v} h_\mathrm{v}, \tag{11}$$

where τ is the relaxation time, λ is the thermal conductivity, θ is the temperature and h_i is the enthalpy of component i per unit mass of component i. For the Cattaneo model the temperature propagation speed is given as

$$c_T = \sqrt{\frac{\lambda}{\rho_\mathrm{l}\tau}}.$$

Eq. (11) can be transformed [19] to the form

$$r = \rho_\mathrm{m} w_\mathrm{m} h_\mathrm{m} - \rho_\mathrm{m} D h_\mathrm{a} \nabla \rho_{\beta\mathrm{a}} - \rho_\mathrm{m} D h_\mathrm{v} \nabla \rho_{\beta\mathrm{v}} \tag{12}$$

or

$$r = \rho_\mathrm{m} w_\mathrm{m} h_\mathrm{m} + \rho_\mathrm{m} D (h_\mathrm{v} - h_\mathrm{a}) \nabla \rho_{\beta\mathrm{a}}. \tag{13}$$

Assuming that

$$\rho e = \rho_\mathrm{s} h_\mathrm{s} + \rho_\mathrm{l} h_\mathrm{l} + \rho_\mathrm{m} h_\mathrm{m} - \rho_\mathrm{m} R_\mathrm{m} \theta, \tag{14}$$

where e is the thermal energy and R is the gas constant, the thermal equations can be expressed as

$$\frac{\lambda}{c_T^2}\frac{\partial^2\theta}{\partial t^2} + \rho c \dot{\theta} = \lambda \nabla^2 \theta + \left(S + \tau \frac{\partial S}{\partial t}\right), \tag{15}$$

where

$$S = -\rho_\mathrm{m}\left[w_\mathrm{m} c_{p\mathrm{m}} + D\left(c_{p\mathrm{v}} - c_{p\mathrm{a}}\right)\nabla\rho_{\beta\mathrm{a}}\right]\nabla\theta - \left[(h_\mathrm{v} - h_\mathrm{a})W_\mathrm{m} - \dot{\overline{\rho_\mathrm{m} R_\mathrm{m} \theta}}\right].$$

2.3. Darcy's law

The velocity of the air–vapor mixture is given by

$$w_\mathrm{m} = -k_\mathrm{D} \nabla p, \tag{16}$$

where k_D is Darcy's coefficient and p is the pressure.

2.4. Thermodynamic relations

Assuming that the vapor and air are ideal gases we have the following relations.

Ideal gas equation for the vapor

$$p_\mathrm{v}\overline{V}_\mathrm{v} = \rho_\mathrm{v} R_\mathrm{v} \theta, \tag{17}$$

where $\overline{V}_i = \rho_i/\rho_{\alpha i}$ represents the volume occupied by component i per unit total volume; and

Ideal gas equation for the air

$$p_\mathrm{a}\overline{V}_\mathrm{a} = \rho_\mathrm{a} R_\mathrm{a} \theta. \tag{18}$$

2.5. Clausius–Clapeyron equation

Since the liquid and vapor are assumed to be in equilibrium,

$$p_\mathrm{v} = p_\mathrm{sat}(\theta) \tag{19}$$

in the presence of liquid water. An analytic expression for p_{sat} is

$$p_{sat}(\theta) = C\theta^{-(B/R_v)}\exp\left(-\frac{A}{R_v T}\right). \tag{20}$$

2.6. State equation

Using the notations $\rho_{\beta i}$ and $\overline{V}_i$, the state equation can be presented as

$$p_v(\varepsilon - \overline{V}_1) = \rho_m \rho_{\beta v} R_v \theta = (1-\rho_{\beta a})\rho_m R_v \theta \tag{21}$$

and

$$p_a(\varepsilon - \overline{V}_1) = \rho_m \rho_{\beta a} R_a \theta, \tag{22}$$

where $\overline{V}_v = \overline{V}_a = (\varepsilon - \overline{V}_1)$ and $\in$ is the porosity.

Combining the above equations we get

$$p(\varepsilon - \overline{V}_1) = \rho_m R_m \theta, \tag{23}$$

where

$$R_m = \rho_{\beta v} R_v + \rho_{\beta a} R_a. \tag{24}$$

3. Example

3.1. Simplified equations for a 1-D axysymetrical problem

One of the simple examples of the thermo-poroelasticity theory presented in this paper is 1-D axysymmetrical problem. In such a case the set of governing equations and suitable boundary conditions in a cylindrical coordinate system of coordinates (r, x) take the form:

Thermal equations

$$\begin{aligned}\frac{\lambda}{c_T^2}\frac{\partial^2\theta}{\partial t^2} + \rho c_p \frac{\partial\dot{\theta}}{\partial t} = \lambda\frac{\partial^2\theta}{\partial x^2} &+ \left\{\frac{1}{r}\frac{\partial}{\partial x}(r\lambda) - \rho_m c_{pm}\right.\\ &\left.\times\left[w_m + \frac{D(c_{pr}-c_{pa})}{c_{pm}}\frac{\partial\rho_{\beta a}}{\partial x}\right]\right\}\frac{\partial\theta}{\partial x}\\ &- \left[(h_v - h_a)W_m - \frac{\partial}{\partial t}(\rho_m R_m \theta)\right]\\ &+ \tau\frac{\partial}{\partial t}\left\{\frac{1}{r}\frac{\partial}{\partial x}(r\lambda) - \rho_m c_{pm}\right.\\ &\left.\times\left[w_m + \frac{D(c_{pr}-c_{pa})}{c_{pm}}\frac{\partial\rho_{\beta a}}{\partial x}\right]\right\}\frac{\partial\theta}{\partial x}\\ &- \left[(h_v - h_a)W_m - \frac{\partial}{\partial t}(\rho_m R_m \theta)\right],\end{aligned} \tag{25}$$

$$\theta(x,0) = \theta_0(x), \tag{26}$$

$$\frac{\partial\theta}{\partial x}(0,t) = 0 \tag{27}$$

and

$$-\lambda\frac{\partial\theta}{\partial x}(L,t) = h\left[\theta(L,t) - \theta_f(t)\right] + f\sigma\left[\theta^4(L,t) - \theta_f^4(t)\right], \tag{28}$$

where L is the radius of the element, θ_f is the environmental temperature, h is the convective heat transfer coefficient, f is the effective shape factor for radiation and σ is the Stefan–Boltzman constant.

Species equations

$$\frac{\partial\rho_{\beta a}}{\partial t} = D\frac{\partial^2\rho_{\beta a}}{\partial x^2} + \left[\frac{1}{\rho_m r}\frac{\partial}{\partial x}(r\rho_m D) - w_m\right]\frac{\partial\rho_{\beta a}}{\partial x} - \frac{\rho_{\beta a}W_m}{\rho_m}, \tag{29}$$

$$\rho_{\beta a}(x,0) = \rho_{\beta a,0}(x), \tag{30}$$

and

$$\frac{\partial\rho_{\beta a}}{\partial x}(0,t) = 0. \tag{31}$$

Continuity equations

$$\frac{\partial\rho_m}{\partial t} + \frac{1}{r}\frac{\partial}{\partial x}(r\rho_m w_m) = W_m, \tag{32}$$

$$w_m(0,t) = 0, \tag{33}$$

$$\frac{\partial\rho_l}{\partial t} = -W_m, \tag{34}$$

and

$$\rho_l(x,0) = \rho_{l,0}(t). \tag{35}$$

3.2. The solution

The solution is obtained by an implicit finite difference technique with a constant grid and variable time step sizes. Since the equations describing heat and mass transfer in porous materials are partial differential equations of parabolic type, the solutions of such equations are well known and can be found in many textbooks on numerical analysis. These data do not provide any special information regarding this paper and have been omitted.

3.3. Temperatures in a 1-D axysymmetrical element

The specific case of a 1-D axysymmetrical structural element with a uniform initial temperature is considered to illustrate the results of the analysis. The element length is 12 cm. The initial moisture content is from 0 to 10.8 cm and the remaining 10.8 to 12 cm is supposed to

Table 1
Thermal parameters used in the study

$c_p = 1040$ J/kg K	$\alpha^\theta = 10 \times 10^{-6}$ 1/K
$D = 2.142 \times 10^{-5}$ m²/s	$\beta = 0.1$
$f = 0.9$	$k_D = 5 \times 10^{-12}$ to ∞ m³ s/kg
$\alpha = 0$ to ∞	$\rho_{\beta a \infty} = 1.0$
$\alpha_D = \infty$	$a = 5.22 \times 10^{-7}$ m²/s
$\lambda = 1.70$ J/s m K	$\in = 0.2$
$A = 3.18 \times 10^6$ J/kg	$\rho = 2400$ kg/m³
$B = 2470$ J/kg K	$\rho_{1,0} = 0.200$ kg/m³
$C = 6.05 \times 10^{26}$ N/m²	

be dry. The thermal parameters used are presented in Table 1. The results will be described in nondimensional values so nondimensional parameters are shown in Table 2. In Fig. 1 the heating curve for the problem analysed is shown. Figs. 2–4 present the nondimensional temperature in the porous element as a function of nondimensional time for the assumed heating curve and various material parameters.

Table 2
Nondimensional parameters used in the study

$B_i = 0$	$Sh = \infty$
$\bar{c}_p = 1.0$	$St = 0.1$
$\overline{D} = 1.0$	$\bar{\alpha} = 1.0$
$\bar{k} = 1.0$	$\rho = 1.0$
$\bar{k}_D = 10$–∞	$\bar{\rho}_{1,0} = 0$–0.1
$L_k = 43$	

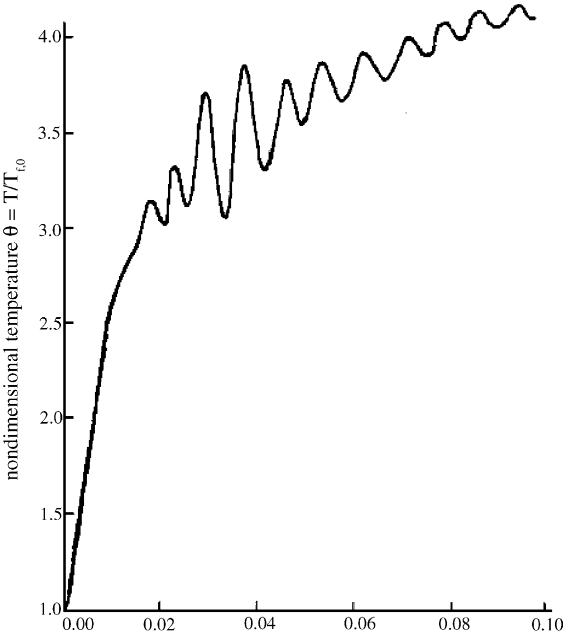

Fig. 1. Heating curve for the problem analysed.

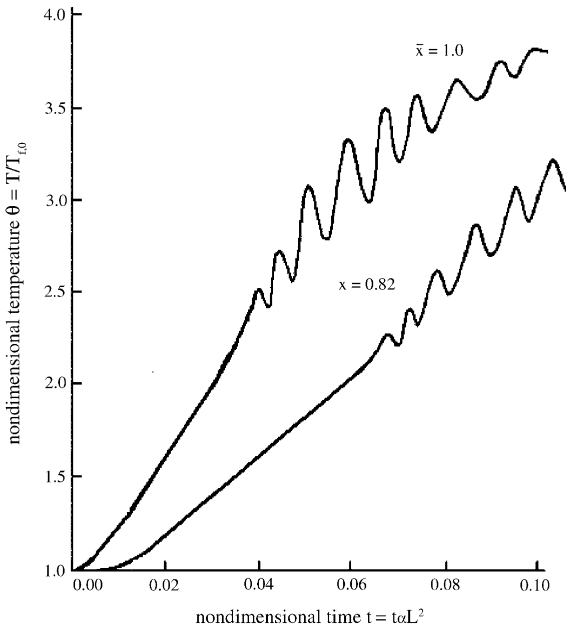

Fig. 2. Nondimensional temperature as a function of nondimensional time for $B_i = 0.6$, $\bar{k}_D = 10$, $\bar{\rho}_{1,0} = 2.9 \times 10^{-2}$.

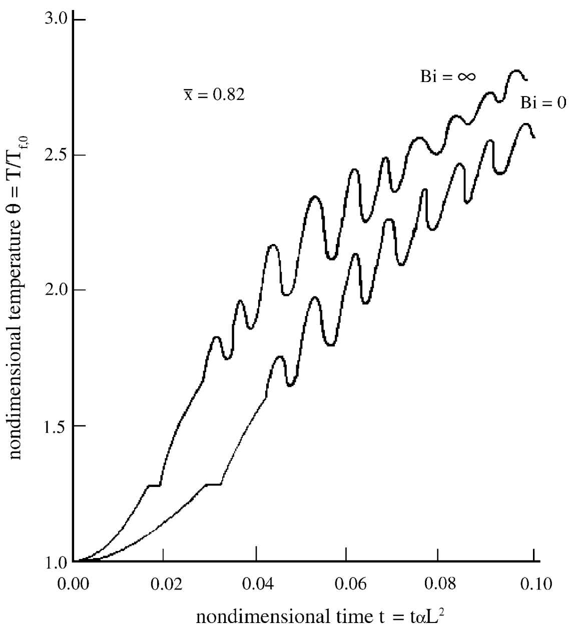

Fig. 3. Nondimensional temperature as a function of nondimensional time for Biot numbers $B_i = \infty$ (a) and $B_i = 0$ (b), $\bar{\rho}_{1,0} = 0.1$ and $\bar{x} = 0.82$ for heating curve as in Fig. 1.

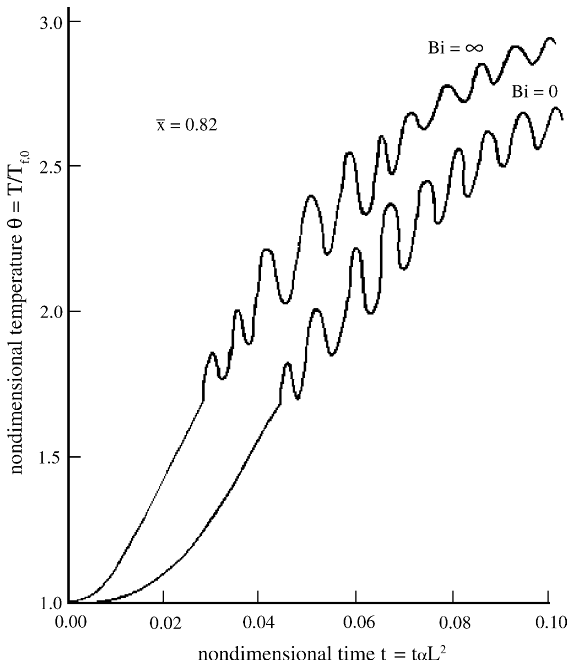

Fig. 4. Nondimensional temperature as a function of nondimensional time for $\bar{\rho}_{1,0} = 0.0$.

4. Concluding remarks

An analysis is developed for thermal waves in a wet porous medium subject to unsteady, nonlinear boundary conditions. The Cattaneo model was introduced. The simplified equations have been solved simultaneously by an implicit finite difference technique.

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